USE OF THE FINITE ELEMENT METHOD FOR SIMULATION OF HEAT GENERATION IN DISC BRAKES

Summary – The aim of this paper was to investigate the temperature fields of the solid disc brake during short, emergency braking. The standard Galerkin weighted residual method was used to discretize the parabolic heat transfer equation. The finite element analysis for two-dimensional model was performed due to the heat flux ratio constantly distributed in circumferential direction. Two types of disc brake with appropriate boundary and initial conditions were developed. Results of calculations for the temperature distributions in radial and axial direction are presented. It was found that presented finite element technique for two-dimensional model with particular assumption in operation and boundary conditions agree well with so far achievements in this field.

1 Introduction

Over decades, frictional heating in brakes and clutches has been investigated by many researches. Temperature rise affected by conversion of large amounts of kinetic energy into heat energy is a complex phenomenon. All characteristics of the process (velocity, pressure, friction coefficient, thermal properties of the materials) vary with time. However, it is important to predict temperature distribution of heat generation during braking and clutch engagement.

Long repetitive braking terms, particularly during mountain descents or high-speed stops (autobahn stop) may cause significant concern. Undesirable effects (low frequency vibrations, fade of the lining with variations of friction coefficient, premature wear) directly affect braking performance. Hence it is essential to know the peak temperatures at the beginning of the design process.

Talati and Jalalifar formulated the problem of two models of heat dissipation in disc brakes: namely macroscopic and microscopic model [1]. In the macroscopic model First Law of Thermodynamics has been taken
into account and for microscopic model various characteristics such as the duration of braking, velocity of the vehicle, material properties or braking pressure have been studied. Green's functions were employed to determine temperature distributions.

Non-axisymmetric model of brake disc system with moving heat source was investigated in paper [2]. Appropriate boundary conditions due to analytical model have been imposed. To solve the problem, a transient FE technique has been used. Numerical estimation reveals that the operating parameters of the braking process significantly influence the disc/pad interface temperature distribution and the maximal contact temperature.

It is important that the analysis is treated as a nonlinear (thermal conductivity and enthalpy for the disc material vary with respect to temperature) [3]. In this study the simulation of the temperature field in the brake disc has been carried out using the finite element method. Both, wear and temperature distribution have been considered.

FE modelling of the heat generation process in a mine winder disc brake is proposed in monograph [4]. Critical value term related to thermal distortions was used to estimate stable range of disc brake operation.

In this study, transient thermal analysis of disc brake utilizing finite element method is developed. Both analytical and numerical investigations are performed. Various boundary and operation conditions in two types of FE models with appropriate material properties [1,2] are established.

2 Real problem

Generally disc brake system in automotive application consists of cast-iron disc which rotates with the wheel, caliper fixed to the steering knuckle and friction material (brake pads). When the braking process occurs, the hydraulic pressure forces the piston and therefore pads and disc brake are in sliding contact. Set up force resists the movement and the vehicle slows down or eventually stops. Friction between disc and pads always opposes motion and the heat is generated. However, friction surface is exposed to the enlarged air flow during braking and the heat is dissipated.

Disc brake. Ordinarily disc brakes are made of gray cast iron and are either solid or ventilated. The ventilated type of disc has vanes or fins to increase surface of heat exchange by convection. Furthermore, higher order of disc brakes have drilled holes. Nowadays a cross-drilled discs are commonly used in motorcycles, racing cars or very high performance road cars. Cross-drilled enables more efficient gas release in the
brake exert. The disc must have limited mass in order to diminish the inertia forces and non-suspending mass.

Pads. Several assumptions should be considered in the case of design process of friction material. It is known that the value of sliding friction depends on the nature of two surfaces which touch each other. Material selection must deal with the coefficient of friction which is supposed to remain constant in the braking process corresponding to wide variety of disc/pad interface temperature. Also wear is vital in case of braking performance.

Caliper. Two types of calipers are commonly used: the floating calipers and the fixed calipers. Depending on the way of operation, the floating caliper has either one or two pistons.

In the floating caliper the piston is located only in one side of the disc. Equal pressure at the same time is distributed on the two inner surfaces of pads by using reaction when the pressure acts piston on the one side of the disc.

The fixed caliper has two pistons in both sides of the disc brake. The equilibrium of pressure at any pad is settled by the single source of the hydraulic pressure partitioned to each canal of the piston.

3 Physical problem

In disc brake system two major parts may be distinguished: rotating axisymmetric disc and immovable non-axisymmetric pad (Fig. 1). The most important function of disc brake system is to reduce velocity of the vehicle by changing the kinetic energy into thermal energy. When the braking process begins total heat is dissipated by conduction from disc/pad interface to adjacent components of brake assembly and hub and by convection to atmosphere in accordance to Newton’s law. The radiation may be neglected due to relatively low temperature and short time of the braking process.

In this paper for validation of proposed finite element (FE) modeling technique, two types of solid disc brake were analyzed (Fig. 2). Type A according to Talati and Jalalifar’s paper [1] and Type B according to Gao and Lin’s paper [2].
Fig. 1. The schematic diagram of a disc brake system

Fig. 2. Models of disc brakes for the transient analysis a) Type A; b) Type B
For both types it has been assumed as follows:

Material properties are isotropic and independent of the temperature;
The real surface of contact between a disc brake and pad in operation is equal to the apparent surface in the sliding contact. Hence pressure is uniformly distributed over all friction surfaces;
The average intensity of heat flux into disc on the contact area equals [5]:

\[ q_p(r,z,t)_{\mid_{\delta_p}} = \frac{\gamma \phi_0}{2\pi} f_p(t) r \omega(t), r_p \leq r \leq R_p, 0 \leq t \leq t_s, \]  

\[ q_p(r,z,t)_{\mid_{\delta_p}} = (1 - \gamma) f_p(t) r \omega(t), r_p \leq r \leq R_p, 0 \leq t \leq t_s, \]  

where: \( \gamma \) is the heat partitioning factor, \( \phi_0 \) is the cover angle of pad, \( f \) is the friction coefficient, \( p \) is the contact pressure, \( \omega \) is the angular velocity, \( t \) is the time, \( t_s \) is the braking time, \( r \) is the radial coordinate, \( z \) is the axial coordinate, \( r_p \) and \( R_p \) are the internal and external radius of the pad. The subscripts \( p \) and \( d \) imply the pad and the disc respectively;
The heat partitioning factor representing the fraction of frictional heat flux entering the disc has the form [6]:

\[ \gamma = \frac{\sqrt{\rho_d c_d K_d}}{\sqrt{\rho_d c_d K_d} + \sqrt{\rho_p c_p K_p}}, \]  

where \( \rho_{d,p} \) is the density, \( c_{d,p} \) is the specific heat and \( K_{d,p} \) is the thermal conductivity;
The frictional heat due to Newton's law has been dissipated to atmosphere on the other surfaces. The heat transfer coefficient \( h \) is constant during braking process;
Because of short braking time and hence relatively low temperature the radiation is neglected.
Two types of single disc have been analyzed with its simplification to symmetrical problem. Therefore one side of the disc has been insulated in both types of the FE model.
In Type A, excluding both the surface of symmetry and the surface of sliding contact with the intensity of heat flux boundary condition, on all remaining surfaces the exchange of thermal energy by convection to atmosphere has been assumed. Furthermore in Type B inner surface of disc was insulated and in the area of sliding contact intensity of the heat flux has been established. The frictional heat due to Newton’s law has been dissipated to atmosphere on the other surfaces.
In Type A the contact pressure \( p \) is given as follows
and the angular velocity $\omega$ is linear in time $t$.

$$
\omega(t) = \omega_0 \left(1 - \frac{t}{t_s^0}\right), \quad 0 \leq t \leq t_s^0
$$

(5)

where: $p_0$ is the nominal pressure, $\omega_0$ is the initial angular velocity, $t_s^0$ is the time of braking with constant deceleration.

The opposite approach is presented in Type B. It is assumed, that the pressure varies with time [7]

$$
p(t) = p_0 \left(1 - e^{-\frac{t}{t_m}}\right), \quad 0 \leq t \leq t_s,
$$

(6)

where: $t_m$ is the growing time. The angular velocity corresponds to pressure (6) and is equal [8]

$$
\omega(t) = \omega_0 \left[1 - \frac{t}{t_s^0} + \frac{t_m}{t_s^0} \left(1 - e^{-\frac{t}{t_m}}\right)\right], \quad 0 \leq t \leq t_s,
$$

(7)

4 Mathematical model

The starting point for the transient analysis of the temperature fields in the disc volume is the parabolic heat conduction equation given in the cylindrical coordinate system which is centered in the axis of disc and $z$ points to its thickness as follows [9]

$$
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k_d} \frac{\partial T}{\partial t},
$$

(8)

$$
\quad r_d \leq r \leq R_d, 0 < z < \delta_d, t > 0,
$$

where $k_d$ is the thermal diffusivity of the disc, $r_d$ and $R_d$ are the internal and external radius of the disc.

The boundary and initial conditions are given as follows

Type A
Use of the finite element...

\[ K_d \frac{\partial T}{\partial z} \bigg|_{z=\delta_d} = \begin{cases} h[T_a - T(r, \delta_d, t)], & r_d \leq r \leq r_p, \ t \geq 0, \\ q_d(r, \delta_d, t), & r_p \leq r \leq R_p, \ 0 \leq t \leq t_i, \end{cases} \quad (9) \]

\[ K_d \frac{\partial T}{\partial r} \bigg|_{r=R_d} = h[T_a - T(R_d, z, t)], \quad 0 \leq z \leq \delta_d, \ t \geq 0, \quad (10) \]

\[ K_d \frac{\partial T}{\partial r} \bigg|_{r=r_d} = -h[T_a - T(r_d, z, t)], \quad 0 \leq z \leq \delta_d, \ t \geq 0, \quad (11) \]

\[ \frac{\partial T}{\partial z} \bigg|_{z=0} = 0, \quad r_d \leq r \leq R_d, \ t \geq 0, \quad (12) \]

\[ T(r, z, 0) = T_a, \quad r_d \leq r \leq R_d, \ 0 \leq z \leq \delta_d, \quad (13) \]

**Type B**

\[ K_d \frac{\partial T}{\partial z} \bigg|_{z=\delta_d} = \begin{cases} h[T_a - T(r, \delta_d, t)], \\ r_d \leq r \leq r_p \land R_p \leq r \leq R_d, t \geq 0; \\ q_d(r, \delta_d, t), & r_p \leq r \leq R_p, t \geq 0, \end{cases} \quad (14) \]

\[ K_d \frac{\partial T}{\partial r} \bigg|_{r=R_d} = h[T_a - T(R_d, z, t)], \quad 0 \leq z \leq \delta_d, \ t \geq 0, \quad (15) \]

\[ \frac{\partial T}{\partial r} \bigg|_{r=r_d} = 0, \quad 0 \leq z \leq \delta_d, \ t \geq 0, \quad (16) \]

\[ \frac{\partial T}{\partial r} \bigg|_{r=R_d} = 0, \quad r_d \leq r \leq R_d, \ t \geq 0, \quad (17) \]

\[ T(r, z, 0) = T_a, \quad r_d \leq r \leq R_d, \ 0 \leq z \leq \delta_d, \quad (18) \]

The above cases are two-dimensional problems for transient analysis.

**5 FE formulation**

The object of this section is to develop approximate time-stepping procedures for axisymmetric transient governing equations. For this to happen, the following boundary and initial conditions are considered

\[ T = T_p \text{ on } \Gamma_r \quad (19) \]
where $T_p$ is the prescribed temperature, $\Gamma_T$, $\Gamma_h$, $\Gamma_q$, are arbitrary boundaries on which temperature, convection and heat flux are prescribed.

In order to obtain matrix form of Eq. (8) the application of standard Galerkin’s approach was used [10]. The temperature was approximated over space as follows

$$T(r, z, t) = \sum_{i=1}^{n} N_i(r, z) T_i(t)$$

where: $N_i$ are shape functions, $n$ is the number of nodes in an element, $T_i(t)$ are time dependent nodal temperatures.

The standard Galerkin’s approach of Eq. (8) leads to the following equation

$$\int_{\Omega} K_d N_i \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} - \rho_s c_s \frac{\partial T}{\partial t} \right] d\Omega = 0$$

Using integration by parts of Eq. (24) we obtain

$$-\int_{\Omega} K_d \left[ \frac{\partial N_i}{\partial r} \frac{\partial T}{\partial r} + \frac{\partial N_i}{\partial z} \frac{\partial T}{\partial z} - N_i \frac{\partial T}{\partial r} \frac{\partial r}{\partial r} + N_i \rho_s c_s \frac{\partial T}{\partial t} \right] d\Omega$$

$$+ \int_{\Gamma} K_d N_i \frac{\partial T}{\partial r} n d\Gamma + \int_{\Gamma} K_d N_i \frac{\partial T}{\partial z} n d\Gamma = 0$$

Integral form of boundary conditions is given

$$\int_{\Gamma} K_d N_i \frac{\partial T}{\partial r} n d\Gamma + \int_{\Gamma} K_d N_i \frac{\partial T}{\partial z} n d\Gamma$$

$$= -\int_{\Gamma} N_i q_d n d\Gamma_q - \int_{\Gamma} N_i h(T - T_0) n d\Gamma_h$$

Substituting Eq. (26) and Eq. (23) to Eq. (25) we obtain
\[
- \int_{\Omega} K_{ij} \left[ \frac{\partial N_i}{\partial r} \frac{\partial N_j}{\partial r} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} - \frac{N_i}{r} \frac{\partial N_j}{\partial r} \right] T_i \, d\Omega \\
- \int_{\Omega} \rho c_d N_i \frac{\partial N_i}{\partial t} T_i \, d\Omega - \int_{\Gamma_i} N_i q_d d\Gamma_i \\
- \int_{\Gamma_i} N_i h(T - T_i) d\Gamma_i = 0
\]  

(27)

where \( i \) and \( j \) represent the nodes.

Eq. (27) can be written in matrix form

\[
[C] \left\{ \frac{\partial T_i}{\partial t} \right\} + [K] [T] = \{ R \}
\]

(28)

where \([C]\) is the heat capacity matrix, \([K]\) is the heat conductivity matrix, and \([R]\) is the thermal force matrix, or

\[
[C_{ij}] \left\{ \frac{\partial T_j}{\partial t} \right\} + [K_{ij}] [T] = \{ R_j \}
\]

(29)

where

\[
[C_{ij}] = \int_{\Omega} \rho c_d N_i N_j \, d\Omega
\]

(30)

\[
[R_j] = -\int_{\Gamma_i} q_d N_i d\Gamma_i + \int_{\Gamma_i} N_i hT_i d\Gamma_i
\]

(31)

\[
[K_{ij}] = \int_{\Omega} K_{ij} \left\{ \frac{\partial N_i}{\partial r} \frac{\partial N_j}{\partial r} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} - \frac{N_i}{r} \frac{\partial N_j}{\partial r} \right\} \, d\Omega \\
+ \int_{\Gamma} hN_i N_j \, d\Gamma
\]

(32)

In order to solve the ordinary differential equation (28) the direct integration method was used. Based on the assumption that temperature \( \{ T \}_i \) and \( \{ T \}_{i+\Delta t} \) at time \( t \) and \( t+\Delta t \) respectively, the following relation is specified

\[
\{ T \}_{i+\Delta t} = \{ T \}_i + \left[ (1-\beta) \left\{ \frac{\partial T}{\partial t} \right\}_i + \beta \left\{ \frac{\partial T}{\partial t} \right\}_{i+\Delta t} \right] \Delta t
\]

(33)

Substituting Eq. (33) to Eq. (28) we obtain the following implicit algebraic equation
\[ ([C] + \beta \Delta t[K])[T]_{t+\Delta t} = ([C] - (1 - \beta)[K][\Delta t][T]_t + (1 - \beta)\Delta t[R]_t + \beta \Delta t[R]_{t+\Delta t} \]

where \( \beta \) is the factor which ranges from 0.5 to 1 and is given to determine an integration accuracy and stable scheme.

Fig. 3. FE models of disc brakes with boundary conditions for the transient analysis a) Type A, b) Type B

The finite element formulation of disc brake with boundary conditions is shown in Fig. 3. Two FE models described below were analyzed using the MD Patran/MD Nastran software package. In the thermal analysis of disc brakes an appropriate finite element division is indispensable. In this study eight-node quadratic elements were used. Type A consists of 235 elements and 810 nodes and Type B 570 elements and 1913 nodes. High order of elements ensure appropriate numerical accuracy.

To avoid inaccurate or unstable results, a proper initial time step associated with spatial mesh size is essential [11].

\[ \Delta t = \Delta x^2 \frac{\rho c_p}{10K_d} \]

where \( \Delta x \) is the time step, \( \Delta x \) is the mesh size (smallest element dimension. In this paper fixed
\( \Delta t = 0.005 \)s time step was imposed.

6 Results and discussion

In this paper temperature distributions in disc brake model without pad have been investigated. It is connected with its sophisticated behaviour and importance of operation. Disc material is subjected to high temperatures action which may cause non-uniform pressure distribution,
thermal distortions, low frequency vibrations. Both convection and conduction have been analyzed. Particularly conduction was considered to be the most important mode of heat transfer.

In order to validate proposed transient numerical analysis two different types of FE models were investigated [1,2]. A transient solution for Type A was performed for operation conditions of constant contact pressure $p_0=3.17\text{MPa}$ and initial angular velocity $\omega_0=88.5\text{s}^{-1}$ during 3.96s of braking process (Fig. 4a). Evolution of the pressure $p$ and angular velocity of the disc $\omega$ for Type B is shown in Fig. 4b. Material properties and operation conditions adopted in the analysis for both types of disc numerical model are given in Tab. 1 and Tab. 2 respectively.
Fig. 4. Evolution of the pressure $p$ and angular velocity $\omega$ during braking process: a) Type A, b) Type B

<table>
<thead>
<tr>
<th>Tabela. 1. Material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disc</td>
</tr>
<tr>
<td>thermal conductivity, $K_d$ [W/mK]</td>
</tr>
</tbody>
</table>
Use of the finite element...

| specific heat, $c_d \text{[J/kgK]}$ | 445 | 900 | 419 | 1465 |
| density, $\rho_d \text{[kg/m$^3$]}$ | 7850 | 2500 | 7228 | 2595 |

Tabela 2. Operation conditions

<table>
<thead>
<tr>
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<tr>
<td></td>
<td>Disc</td>
<td>Pad</td>
</tr>
<tr>
<td>inner radius, $r_{i,d} \text{[mm]}$</td>
<td>66</td>
<td>76.5</td>
</tr>
<tr>
<td>outer radius, $R_{o,d} \text{[mm]}$</td>
<td>113.5</td>
<td>128</td>
</tr>
<tr>
<td>cover angle of pad, $\Delta \theta$</td>
<td>64.5</td>
<td>64.5</td>
</tr>
<tr>
<td>disc thickness $d_{d} \text{[mm]}$</td>
<td>5.5</td>
<td>6</td>
</tr>
<tr>
<td>initial velocity $\Delta \omega \text{[s$^{-1}$]}$</td>
<td>88.5</td>
<td>88.5</td>
</tr>
<tr>
<td>time of braking, $t_s \text{[s]}$</td>
<td>3.96</td>
<td>4.274</td>
</tr>
<tr>
<td>pressure $p_0 \text{[MPa]}$</td>
<td>3.17</td>
<td>3.17</td>
</tr>
<tr>
<td>coefficient of friction $f$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>heat transfer coefficient $h \text{[W/m$^2$K]}$</td>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>initial temperature $T_0 \text{[°C]}$</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>ambient temperature $T_a \text{[°C]}$</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>time step $\Delta t \text{[s]}$</td>
<td>0.005</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Fig. 5a shows disc surface temperature distribution for transient numerical computation (Type A) at different radial distances. As it can be seen values of temperature increase with radial distances. The highest temperature of brake exert occurs at 113.5mm of radial position and $t=3.025$s of time. Temperature distribution corresponds intermediately to the intensity of heat flux, which rises with time until the value of velocity and pressure product attains highest, critical value. Hence temperature indirectly increases with time and decreases when the intensity of heat flux $q_d$ descents. The slope $\frac{dT}{dt}$ of plots $r=75.5\text{mm}$, $r=80\text{mm}$, $r=90\text{mm}$, $r=100\text{mm}$, $r=113.5\text{mm}$ decreases with time. It agree well with Talati and Jalalifar’s paper [1] with distinction to values of temperatures. In this paper the highest temperature of disc area, which occurs during emergency braking achieves $227.90^0\text{C}$.
Fig. 5. Evolution of the disc temperature on the friction surface for different values of the radial position: a) Type A, b) Type B

Fig. 5b shows disc temperature surface variations along radial direction obtained in numerical computation for Type B. In opposite to constant pressure at the disc/pad interface, in this case pressure differs with time (Fig. 4b.). Also angular velocity has been assumed to be nonlinear. As it can be seen temperature at inner disc surface ($r=52$ mm) has a constant value of $20^0C$. It corresponds to boundary conditions, where surface was insulated. Maximum temperature rise up to $280.9^0C$ at $113$mm of radial position and $3.49$s of time.

In Fig. 6a disc temperature in Type A at $r=113.5$mm and at different axial positions is illustrated. Symmetry in axial direction has been assumed. Hence plots from $z=0$mm to maximum thickness of the disc are shown. At the initial period of braking process the value of maximum temperature appears at the disc/pad interface ($z=5.5$mm). There is a
tendency to convergence of temperature at different axial positions at the end of braking. It is connected with alignment of temperatures in disc area in subsequent stage of the process when the intensity of heat flux descents. Temperature of plots $z=4.4\text{mm}$, $z=5.5\text{mm}$ rises with time to 3.47s and 3.025s respectively.

In Fig. 6b temperature distribution at $r=113\text{mm}$ at different axial locations is shown. As it can be seen temperature of plots $z=4\text{mm}$, $z=5\text{mm}$, $z=6\text{mm}$ increases with time to 4s, 3.74s and 3.44s respectively and then decreases while temperature of plots $z=0\text{mm}$, $z=1\text{mm}$, $z=2\text{mm}$, $z=3\text{mm}$ constantly grows.

![Fig. 6. Evolution of the disc temperature at different axial distances and at radial position: a) $r=113.5\text{mm}$ (Type A) b) $r=113\text{mm}$ (Type B)](image-url)
7 Conclusion

In this paper transient thermal analysis of disc brake in single braking action was performed. To obtain the numerical simulation parabolic heat transfer equation for two-dimensional model was used. The results show that both evolution of rotating speed of disc and contact pressure with specific material properties intensely effect disc brake temperature fields in the domain of time. Proposed transient FE modeling technique of two types of disc brakes agree well with papers [1,2]. An instant pressure operation at the disc/pad interface (Type A) pronouncedly implies temperature growth at initial period of brake exert. More slightly temperature rise in Type B has been noticed. The highest temperature occurs approximately at 3s, 3.5s into the braking process for the period of 3.96s, 4.274s time in Type A and Type B respectively. The present paper is a preliminary of subsequent investigation with nonlinear variations of applied thermal characteristics.

References

Streszczenie - Celem pracy było zbadanie pól temperatury litej tarczy hamulcowej podczas krótkiego hamowania bezpieczeństwa. Do dyskretyzacji parabolicznego równania przewodnictwa ciepła wykorzystano metodę resiudów ważonych Galerkin’a. Przeprowadzono symulację MES dla modelu dwuwymiarowego z uwzględnieniem jednorodnego rozkładu mocy sił tarcia w kierunku obwodowym. Opracowano dwa typy hamulca tarczowego z odpowiednimi warunkami początkowymi i brzegowymi. Przedstawiono wyniki obliczeń rozkładów pól temperatury w kierunku promieniowym oraz osiowym. Prezentowana aplikacja metody elementów skończonych dla modelu dwuwymiarowego ze specjalnymi założeniami parametrów analizy wraz z warunkami brzegowymi zgadza się z dotychczasowymi osiągnięciami w obszarze omawianego zagadnienia.