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TOPOLOGY OPTIMIZATION OF MECHANICAL STRUCTURES USING ARTIFICIAL IMMUNE SYSTEM

Summary – The paper deals with an application of the artificial immune system (AIS) and the finite element method to the optimization problems of 2-D, 3-D and the combination of 2-D and 3-D structures. The optimization method concerns the simultaneous optimization of topology, shape, and material. This approach is based on the mechanism discovered in biological immune systems. The main advantage of the AIS is the fact that this approach does not need any information about the gradient of the fitness function and gives a strong probability of finding the global optimum. The numerical examples demonstrate that the method based on evolutionary computation is an effective technique for solving computer aided optimal design.

1 Introduction

The present researches are based on the application of the artificial immune system and the finite element method to the optimization problems of 2-D, 3-D and the combination of 2-D and 3-D structures. The optimization method concerns the simultaneous optimization of topology, shape, and material of the structure. This work is an extension of previous researches of Burczyński, Poteralski and Szczepanik concerning evolutionary optimization problems [3,4,7,8,9]. The main feature of these methods is to simulate biological processes. The artificial immune system is based on the mechanism discovered in biological immune systems. The main advantage of artificial immune

system is the fact that these approach does not need any information about the gradient of the fitness function and gives a strong probability of finding the global optimum. The main drawback of the approach is the long time of calculations. The fitness function is calculated for each B-cell in each iteration by solving a boundary-value problem by means of the finite element method (FEM). In order to solve the optimization problem the fitness function, design variables and constraints should be formulated.

2 The formulation of the problem

Consider a structure which, at the beginning of an immune process, occupies a domain $\Omega_0 \left(in\ E^d, d = 2\ or\ 3 \right)$, bounded by a boundary Γ_0. The domain Ω_0 is filled by elastic homogeneous and isotropic material of a Young's modulus E_0, a mass density ρ_0 and a Poisson ratio ν. The structures are considered in the framework of the linear theory of elasticity. During the immune optimization process the domain Ω_t, its boundary Γ_t and the field of mass densities $\rho(X) = \rho_t, (X) \in \Omega_t$ can change for each iteration *t* (for *t*=0, ρ_0=const). The immune process proceeds in the environment in which the structure fitness is described by the minimization of the mass of the structure

$$J = \int_{\Omega} \rho d\Omega \tag{1}$$

with constraints imposed on displacements of the structure

$$\left| u(x, y, z) \right| \le u^{ad}, (x, y, z) \in \Omega \tag{2}$$

or with constraints imposed on equivalent stresses of the structure

$$\sigma_{eq}(x, y, z) \le \sigma^{ad}, (x, y, z) \in \Omega \tag{3}$$

In order to solve the formulated problems the finite element models of 2-D, 3-D and combination of 2-D and 3-D structures are considered. The 2-D structure domain Ω_{2D} is divided into shell finite elements and the 3-D structure domain is divided into solid finite elements. The additional elements (super-elements) must be applied for modeling shell-to-solid transition (a shell element's translational and rotational degrees of freedom to a solid element's translational degrees of freedom). In order to solve direct problems for combination of 2-D and 3-D elastic

structures the professional program MSC NASTRAN is used. MSC NASTRAN enables application of the proper super elements which are called RSSCON elements [6]. When using the RSSCON capability, the shell element mesh must line up with the solid element mesh so that there is an exact element-to-element correspondence (Fig. 1a). RSSCON generates a multipoint constraint, which puts the shell degrees of freedom in the dependent set. The three translational degrees of freedom and the two rotational degrees of freedom of the shell edge are connected to the three translational degrees of freedom of the upper and lower solid edge. Poisson's ratio effects and temperature loads are modeled correctly.

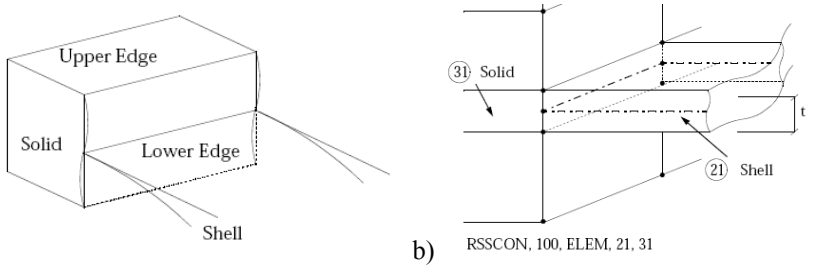

Figure 1: Shell-to-solid transition: a) clamped connection of a shell to a solid, b) best modeling practice for RSSCON

The best modeling practice is illustrated in Fig. 1b. The height of the connected solid element should be chosen equal to the thickness of the shell. If the height of the connected solid element is much larger than the thickness of the shell element, then the connection modeled with RSSCON will be stiffer than the continuum model. Additionally, in a mesh where shell grid points are identical or coincide with solid grid points, the RSSCON elements may model a connection that is too stiff.

3 The idea of immune optimization

The distribution of mass density $\rho(X),\ (X)\in\Omega_t$ (Fig. 2) in the structure is described by a surface $W_\rho(X),(X)\in H^2$ (for 2-D) and a hyper surface $W_\rho(X),(X)\in H^3$ (for 3-D). The surface (hyper surface) $W_\rho(X)$ is stretched under $H^d\subset E^d,(d=2,3)$ and the domain Ω_t is included in H^d, i.e. $\left(\Omega_t\subseteq H^d\right)$.

The shape of the surface (hyper surface) $W_\rho(X)$ is controlled by B-cell receptors d_j, j=1,2,…,G, which create a B-cell

$$ch = \left\langle d_1, d_2, ..., d_j, ..., d_G \right\rangle \quad (4)$$

$$d_j^{\min} \le d_j \le d_j^{\max} \quad (5)$$

where $d_j^{\min}$, $d_j^{\max}$ - are minimum and maximum values of the B-cell receptor, respectively.

B-cell receptors are the values of the function $W_\rho(X)$ in the control points $(X)_j$ of the surface (hyper surface), i.e. $d_j = W_\rho\left[(X)_j\right], \mathrm{j} = 0,1,2,...,G$.

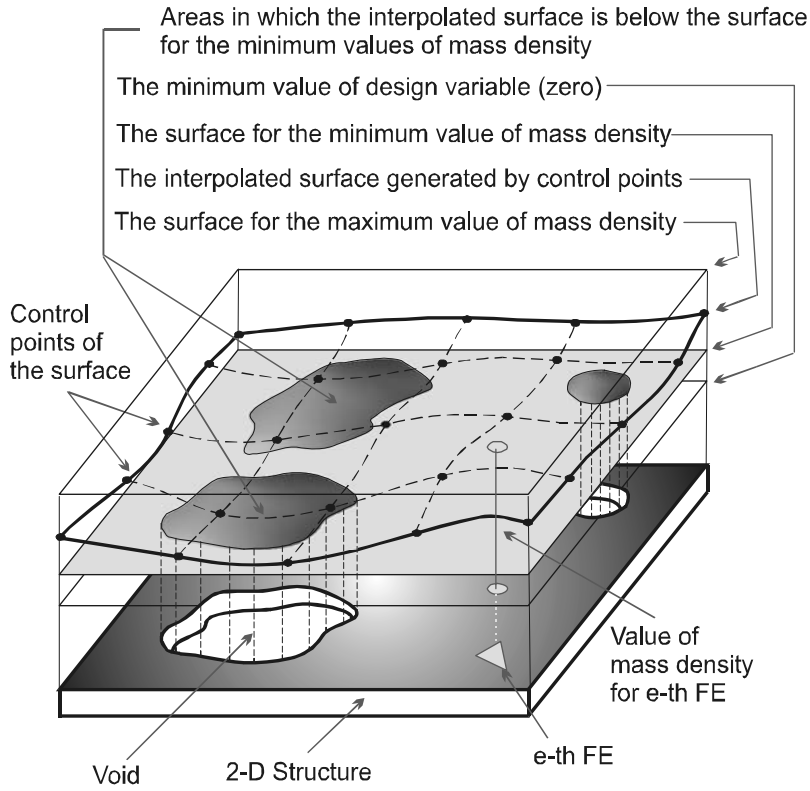

Figure 2: The illustration of the idea of immune optimization for a 2-D structure

The finite element method [11] is applied in analysis of the structure. The domain Ω of the structure is discretized using the finite elements, $\Omega = \bigcup_{e=1}^{E} \Omega_e$.

The assignation of the mass density to each finite element $\Omega_e, e = 1,2,...,E$ is performed by the mappings:

$$\rho_e = W_\rho\left[(X)_e\right], (X)_e \in \Omega_e, e = 1,2,...,E \tag{6}$$

It means that each finite element can have different mass density. When the value of the mass density for the e-th finite element is included in:

- the interval $0 \le \rho_e < \rho_{\min}$, the finite element is eliminated and the void is created,
- the interval $\rho_{\min} \le \rho_e < \rho_{\max}$, the finite element remains.

In the next step the Young's modulus for the e-th finite element is evaluated using the following equation

$$E_e = E_{\max}\left(\frac{\rho_e}{\rho_{\max}}\right)^r \tag{7}$$

where:

$E_{\max}, \rho_{\max}$ - Young's modulus and mass density for the same material, respectively,

r – parameter which can change from 1 to 9.

The dependence between Young's modulus and mass density in the topology optimization was proposed for the first time by Bendsøe [2]. For the topology optimization of 2-D structures the expression (7) was applied by Kutyłowski [5]. By means of the proposed method, the material properties of finite elements are changed and some of them are eliminated. As a result the optimal shape, the topology and the material of the structures are obtained.

4 Parameterization

Parameterization is the key stage in the structural optimization. The great number of design variables causes that the optimization process is not effective. A connection between design variables (B-cell receptors) and number of finite element leads to poor results. The better results can be obtained when the surface (or hyper surface) of mass density distribution is interpolated by suitable number of values given in control points $(X)_j$. This number, on the one hand, should provide the good interpolation, and on the other hand the number of design variables should be small. The interpolation procedure based on some nodes overlapping selected FEM nodes has been introduced. This procedure (Tab. 1) is based on the analysis of the neighbourhoods of the individual nodes and enables introduction of optional number of the control points in any nodes of the finite element mesh.

Table 1: Interpolation procedure in the optimization method

```
Load nodes i=1,2,...,N and elements e=1,2,…,E
For i=1,2,...,N  load the initial vector of interpolation parameters
For k=0,1,2,...,K      „k – step of iteration”
{
For i=1,2,...,N    „for all the nodes”
{
If T_i=0                              „i-th node does not contain a control point”
{
For  l=1,2,...,M                      „for all neighbouring nodes of i-th node”
Calculate max(p_l); Calculate min(p_l)
Calculate p_ik+1=1/2[max(p_lk)+ min(p_lk)]
}
If T_i=1     p_ik+1= d_j, j=j+1        „i-th node contains a control point”
}
}
```

5 Additional procedures

Two different additional procedures have been introduced: the additional procedure aiding the topology optimization, the smoothing procedure. The work of the additional procedure aiding the topology optimization is base on the material elimination (in the regions with low stresses) to the moment when the admissible limit of the stresses is exceeded. The final structure obtained after the optimization process has rough external and internal boundary. In order to get the smooth shape of the boundaries, the procedure which smoothes them has to be used.

6 Operations performed for a single B-cell

The artificial immune system works on the group of B-cells. The operations described above are performed for a single B-cell from the population and lead to the evaluation of the fitness function value (Fig. 3).

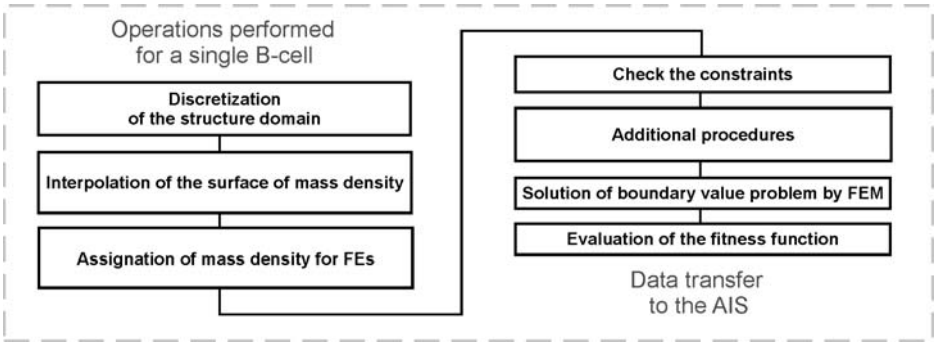

Figure 3: Operation scheme performed for a single B-cell

7 The artificial immune system

The artificial immune systems [1] are developed on the basis of a mechanism discovered in biological immune systems. An immune system is a complex system which contains distributed groups of specialized cells and organs. The main purpose of the immune system is to recognize and destroy pathogens - funguses, viruses, bacteria and improper functioning cells. The lymphocytes cells play a very important role in the immune system. The lymphocytes are divided into several groups of cells. There are two main groups B and T cells, both contains some subgroups (like B-T dependent or B-T independent). The B cells contain antibodies, which could neutralize pathogens and are also used to recognize pathogens. There is a big diversity between antibodies of the B cells, allowing recognition and neutralization of many different pathogens. The B cells are produced in the bone marrow in long bones. A B cell undergoes a mutation process to achieve big diversity of antibodies. The T cells mature in thymus, only T cells recognizing non self cells are released to the lymphatic and the blood systems. There are also other cells like macrophages with presenting properties, the pathogens are processed by a cell and presented by using MHC (Major Histocompatibility Complex) proteins. The recognition of a pathogen is performed in a few steps (Fig. 4). First, the B cells or macrophages present the pathogen to a T cell using MHC (Fig. 4b), the T cell decides if the presented antigen is a pathogen. The T cell gives a chemical signal to B cells to release antibodies. A part of stimulated B cells goes to a lymph node and proliferate (clone) (Fig. 4c). A part of the B cells changes into memory cells, the rest of them secrete antibodies into blood. The secondary response of the immunology system in the presence of known pathogens is faster because of memory cells. The

memory cells created during primary response, proliferate and the antibodies are secreted to blood (Fig. 4d). The antibodies bind to pathogens and neutralize them. Other cells like macrophages destroy pathogens (Fig. 4e). The number of lymphocytes in the organism changes, while the presence of pathogens increases, but after attacks a part of the lymphocytes is removed from the organism.

The artificial immune systems (AIS) Balthrop J. etall (2002), de Castro L. N and Timmis J. (2003), de Castro L. N. and Von Zuben F. J. (2002) take only a few elements from the biological immune systems. The most frequently used are the mutation of the B cells, proliferation, memory cells, and recognition by using the B and T cells. The artificial immune systems have been used to optimization problems by de Castro L. N. and Von Zuben F. J. (2002), classification and also computer viruses recognition Balthrop J. et all (2002). The cloning algorithm Clonalg presented by von Zuben and de Castro uses some mechanisms similar to biological immune systems to global optimization problems. The unknown global optimum is the searched pathogen. The memory cells contain design variables and proliferate during the optimization process. The B cells created from memory cells undergo mutation. The B cells evaluate and better ones exchange memory cells. In Wierzchoń S. T. (2001) version of Clonalg the crowding mechanism is used - the diverse between memory cells is forced. A new memory cell is randomly created and substitutes the old one, if two memory cells have similar design variables. The crowding mechanism allows finding not only the global optimum but also other local ones. The presented approach is based on the Wierzchoń S. T. (2001) algorithm [10], but the mutation operator is changed. The Gaussian mutation is used instead of the nonuniform mutation in the presented approach.

The Fig. 5. presents the flowchart of an artificial immune system. The memory cells are created randomly. They proliferate and mutate creating B cells. The number of nc clones created by each memory cell is determined by the memory cells objective function value. The objective functions for B cells are evaluated. The selection process exchanges some memory cells for better B cells. The selection is performed on the basis of the

geometrical distance between each memory cell and B cells (measured by using design variables). The crowding mechanism removes similar memory cells. The similarity is also determined as the geometrical distance between memory cells. The process is iteratively repeated until the stop condition is fulfilled. The stop condition can be expressed as the maximum number of iterations.

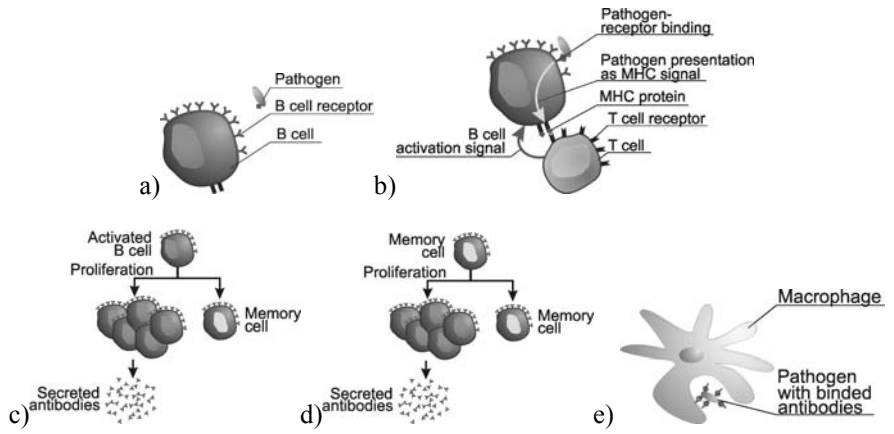

Figure 4: An immune system, a) a B cell and pathogen, b) the recognition of pathogen using B and T cells, c) the proliferation of activated B cells, d) the proliferation of a memory cell – secondary response, e) pathogen absorption by a macrophage

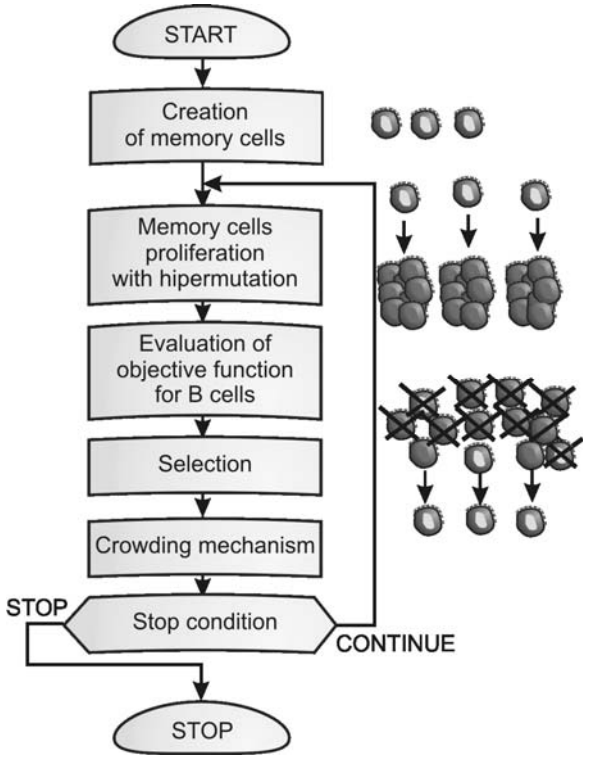

Figure 5: An artificial immune system

8 Numerical examples

Three numerical examples are considered, i.e. the optimization of the shape, the topology and the density of:

- a plate in plane stress (example 1),
- a solid body (example 2),
- a shell-solid structure (example 3),

by the minimization of the mass functional and with the stress or displacement constraint. The structures are considered in the framework of the theory of elasticity. The results of the examples are obtained by using an optimization method based on the artificial immune system with the parameters included in Tab 2.

Table 2: The parameters of artificial immune system

the number of memory cells	8
the number of the clones	4
crowding factor	25%
Gaussian mutation	20%

10.1. Example 1 – The optimization of the shape, the topology and the density of a plate in plane stress

A retriangular 2-D structure (plane stress), of dimensions 100 $\times$ 200 mm, loaded with the concentrated force P in the center of the lower boundary and fixed on the bottom corners is considered. In order to obtain the symmetrical results a half of the structure has been analyzed. The input data to the optimization program and the parameters of the artificial immune system are included in Tab. 3 and 2, respectively. The geometry, the distribution of the control points of the interpolation surface is shown in the Fig. 6a and 6b, respectively. The results of the optimization process are presented in the Fig. 7.

Table 3. The input data to the optimization task of a plate in plane stress

σ^{ad} [Mpa]	the thickness [mm]	σ_{min} ; p [Mpa]	P [kN]	range of ρe [g/cm3]
80.0	4.0	1.0 ; 1.0	2.0	$7.3 \leq \rho e < 7.5$ elimination $7.5 \leq \rho e \leq 7.86$ existence

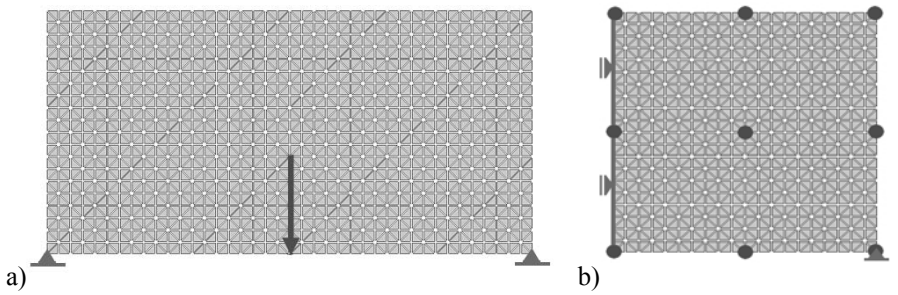

Figure 6: The plate (example 1); a) the geometry; b) the distribution of the control points of the interpolation surface

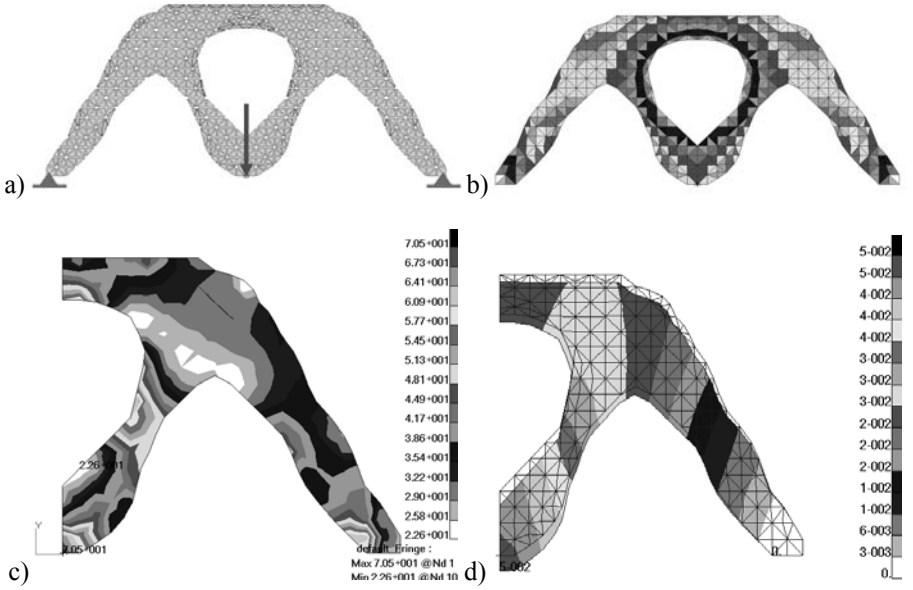

Figure 7: The results of the immune optimization of the plate: a) the solution of the optimization task; b) the map of mass densities; c) the map of stresses; d) the map of the displacement

10.2. Example 2 – The optimization of the shape, the topology and the density of a solid body

A 3-D structure with dimensions and loading is presented in the Fig. 8a and 8b. The input data to the optimization program are included in Tab. 4. The geometry, the distribution of the control points of the interpolation hyper surface is shown in the Fig. 8c. The results of the optimization process are presented in the Fig. 9 and Fig 10.

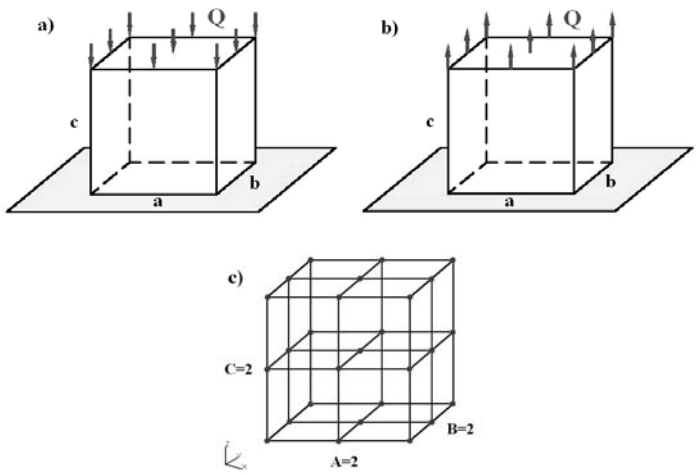

Figure 8: Two cases of loading with the hyper surface a) first case (compression), b) second case (tension), c) the distribution of the control points of the interpolation hyper surface

Table 4: Input data - geometry and loading

Dimensions [mm]			Loading Q	
a	b	c	compression	tension
100	100	100	-36.3 [KN]	36.3 [KN]

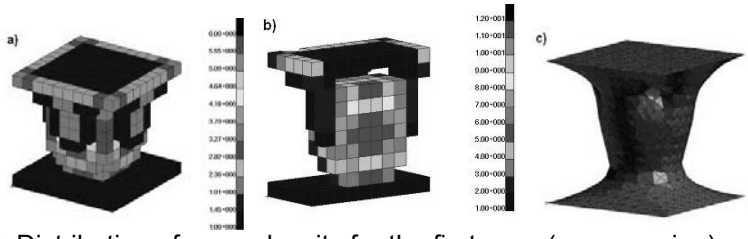

Figure 9: Distribution of mass density for the first case (compression) a), b) structure after 50 iteration (the best solution) c) structure after smooth

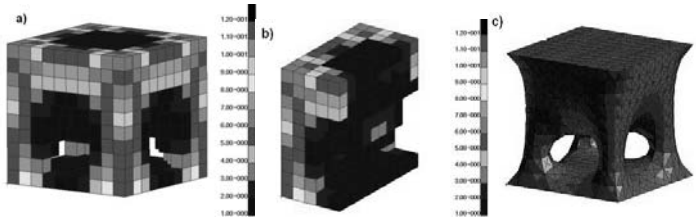

Figure 10: Distribution of mass density for the second case (tension) a), b) structure after 50 iteration (the best solution) c) structure after smooth

10.3. Example 3 – The optimization of the shape, the topology and the density of a shell-solid structure

The structure is stiffly supported on the bottom boundary of the solid body. The upper surface is loaded with pressure. The geometry, the boundary conditions and the distribution of the control points of the interpolation hyper surface are presented in the Fig. 11. The structure is discretized by tetrahedron finite elements for 3-D structure and by triangular elements for 2-D structure. The special elements which combine 2-D finite element with 3-D finite element (MSC NASTRAN RSSCON shell-to-solid element connector) are used. The input data to the optimization task and the parameters of the artificial immune system are included in the Tab. 5 and 2, respectively. The results of the optimization process are presented in the Fig. 12.

Table 5. The input data to the optimization task of a plate in plane stress

σ^{ad} [Mpa]	the thickness [mm]	σ_{min} ; p [Mpa]	p [MPa]	range of ρe [g/cm3]
150.0	15.0	2.0 ; 2.0	3.0	$7.3 \leq \rho e < 7.5$ elimination $7.5 \leq \rho e \leq 7.86$ existence

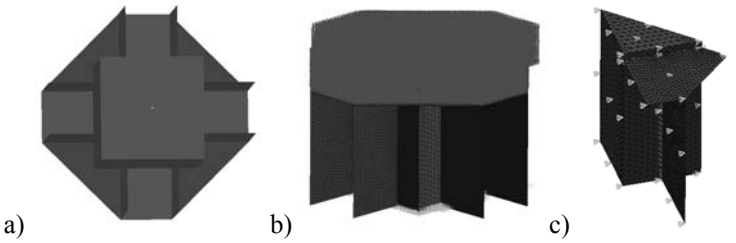

Figure 11: The shell-solid structure (example 3); a) the geometry; b) the boundary condition; c) the distribution of the control points of the interpolation hyper surface

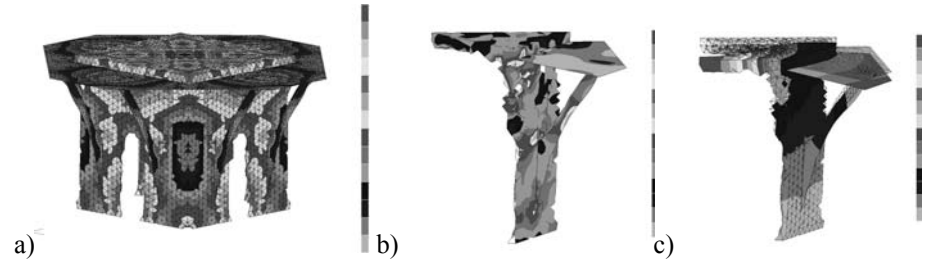

Figure 12: The results of the immune optimization of the shell-solid structure: a) the solution of the optimization task; (the map of mass densities); c) the map of stresses; d) the map of the displacement

9 Conclusions

An effective tool of optimization of the structures has been presented. Using this approach shape, topology and material optimization is performed simultaneously. The important feature of this approach is the strong probability of finding the global optimal solutions received by the implementation of the artificial immune system. The described approach is free from limitations connected with classic gradient optimization methods. Coupling the finite element method and the artificial immune system gives an effective and efficient alternative optimization tool, which enables solving a large class of the optimization problems of mechanical structures. The main feature of the proposed optimization method is the immune distribution of the material in the construction changing its material properties. This process leads to the elimination of the part of material from the construction and as a result the new shape and the topology of the construction emerges. The application of interpolation surfaces (hyper surface) reduces the number of the design variables and shortens the time of the computation. The application of the professional program of the finite element method MSC NASTRAN in this method enables the optimization of the complex mechanical systems. The numerical examples confirm the efficiency of the proposed optimization method and demonstrate that the method based on immune computation is an effective technique for solving computer aided optimal design problems.

10 Acknowledgements

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OPTYMALIZACJA TOPOLOGICZNA UKŁADÓW MECHANICZNYCH PRZY UŻYCIU SZTUCZNEGO SYSTEMU IMMUNOLOGICZNEGO

Streszczenie **-** Artykuł dotyczy zastosowania sztucznego systemu immunologicznego (SSI) i metody elementów skończonych do optymalizacji układów 2-D, 3-D oraz połączonych układów 2-D i 3-D. Metoda optymalizacji dotyczy równoczesnej optymalizacji topologii, kształtu oraz własności materiałowych układu. Podejście to bazuje na mechanizmach zaobserwowanych w biologicznych systemach immunologicznych. Główną zaletą SSI jest fakt, że podejście to nie wymaga informacji o gradiencie funkcji przystosowania i daje duże prawdopodobieństwo znalezienia optimum globalnego. Przykłady

numeryczne przedstawiają, że metoda bazująca na obliczeniach immunologicznych jest efektywnym narzędziem komputerowego wspomagania optymalnego projektowania.