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ON CORRECTNESS OF THE BASICS OF PALUMBO AND PENNISI'S MEANS OF HARMONIC DISTORTION CALCULATION IN ANALOG INTEGRATED CIRCUITS

Summary - It is shown that the method of Palumbo and Pennisi of harmonic distortion calculation is nothing else but a highly specialized variant of the analysis based on the Volterra series. Their power series-like description of a nonlinear system turns out to be a truncated Volterra series which is valid for only one type of input signals. Moreover, simple derivation of their basic but unclear formulae is presented here.

1 Introduction

The authors of [1] and [2] suggest that their method of high-frequency harmonic distortion calculation applied to mildly nonlinear amplifiers (possessing a linear or nonlinear feedback loop) is an approach different from any one based on the use of the Volterra series. We show here that just the opposite is true. It will be evident from our analysis that this method is nothing else than a highly specialized (to calculations of exclusively harmonic distortion) approach based on the above series. In particular, we will show that the basic expression

$$x_o = a_1\left(j\omega\right)x_i + a_2\left(j\omega\right)x_i^2 + a_3\left(j\omega\right)x_i^3 \tag{1}$$

used in [1] (see Fig. 2 or eq. 15 there) and in [2] (see eq. 5 there) to describe a weakly nonlinear dynamic (possessing memory) system (circuit, amplifier) represents a (truncated) Volterra series in a specific form. It can be viewed, as we will show, as an associated system model (associated with the original one which is described by an original Volterra series). In (1) and in what follows, we use the similar notation as assumed in [1] and [2]. That is x_i and x_o in (1) represent the input and output signal, respectively, at the weakly nonlinear system. The angular frequency is denoted by $\omega = 2\pi f$ with *f* meaning the usual frequency variable, and $j = \sqrt{-1}$. Furthermore, the frequency-dependent coefficients $a_1(j\omega)$, $a_2(j\omega)$, and $a_3(j\omega)$ are the first three coefficients in this power series-like description of a mildly nonlinear dynamic system. In other words, after Palumbo and Pennisi [1], they characterize a slightly nonlinear amplifier in the frequency domain, for higher frequencies. Note also, and this is very important, that the description (1) is specific in the sense that it is valid only for the input signals of the form $x_i(t) = A_i \exp(j\omega t)$ where A_i is a real number and means the amplitude of this harmonic signal, and *t* is a time variable.

In [1] and [2], a simpler case of a weakly nonlinear system without memory is also considered. Then, its input-output characteristic is expressed, in the time domain, by a truncated power series of the form

$$x_o = a_1 x_i + a_2 x_i^2 + a_3 x_i^3 \tag{2}$$

where the coefficients a_1, a_2, and a_3 are now real numbers, and the input signal $x_i(t)$ is any function (in this case, eventually, we can have problems, as reported in [3], with input signals being Dirac impulses). Obviously, $x_i(t)$ in (2) is not restricted, exclusively, to the form $x_i(t) = A_i \exp(j\omega t)$.

The authors of [1] and [2] exploit the fact that the form of (1) is similar to that of (2); and this is one of the essential points in their method of calculation of high frequency harmonic distortion. The above similarity, however, cannot lead to the conclusion that because (2) holds therefore (1) must also hold. In other words, the description of a weakly nonlinear dynamic system in the form given by expression (1) seems to be assumed a priori in [1] and [2]. It is of course true under the restrictions given (regarding the form of the input signals applied to it). Here, using the Volterra series, we present the detailed derivation of (1).

We pay also the reader's attention to the fact that the nonlinear system model given by (1) is a kind of a mixed one: the description of the signals ($x_i = x_i(t)$ and $x_o = x_o(t)$) occurring in it is in the time domain, but the description of its parameters (the coefficients $a_1(j\omega)$, $a_2(j\omega)$, and $a_3(j\omega)$) is done in the frequency domain. In opposite to this is the model given by (2) which is purely time domain based.

Moreover, another point in [1] and [2] causes also some confusion. That is the suggested way of finding the response of a weakly nonlinear dynamic system excited by an input signal

$$\mathrm{Re}\left\{A_i \exp\left(j\omega t\right)\right\} = A_i \cos\left(\omega t\right) \tag{3}$$

when its response to another input signal $A_i \exp\left(j\omega t\right)$ (auxiliary one) is known from the calculation with the use of the associated model given by (1). The symbol $\mathrm{Re}\{\cdot\}$ in (3) denotes the operation of taking the real value of a complex number.

In [1] and [2], the way suggested there is not applied, only the final results are given. They are correct.

This point is tackled in detail here because the way of calculation suggested in [1] and [2] leads to incorrect results. It is explained why. The correct way of performing the calculations mentioned is presented in this paper.

2 Derivation of (1)

The Volterra series for nonlinear dynamic (possessing memory) systems of continuous time *t* is defined as [4], [5]

$$y(t) = y^{(1)}(t) + y^{(2)}(t) + y^{(3)}(t) + \ldots = \int_{-\infty}^{\infty} h^{(1)}(\tau)x(t-\tau)d\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 +$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(3)}(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + .. \; . \tag{4}$$

In (4), the output and input signals at the system are denoted by $y(t)$ and $x(t)$, respectively. Moreover, the components of $y(t)$ shown, that is the terms $y^{(1)}(t)$, $y^{(2)}(t)$, $y^{(3)}(t)$, and so on, are called the system partial responses of the corresponding orders: first, second, third, and of higher orders, accordingly. Furthermore, $h^{(1)}(t)$, $h^{(2)}(t_1,t_2)$, $h^{(3)}(t_1,t_2,t_3)$, and so on, are, respectively, the first order, second order, third order, and so on, nonlinear impulse responses of the system considered. Finally, note that $y^{(1)}(t)$ and $h^{(1)}(t)$ are also called its linear partial response and its linear impulse response, respectively.

It is assumed in the literature (for more discussion see, for example, [1] and [2]) that a system which is described by the converging Volterra series (4) behaves weakly nonlinear if its response can be approximated accurately enough (for a set of input signal amplitudes and frequencies considered) through the first three terms of this series. That is through $y^{(1)}(t)$, $y^{(2)}(t)$, and $y^{(3)}(t)$. Moreover, it follows from the discussion presented in [1] and [2] that the class of mildly nonlinear systems considered there is exactly the same that that defined above. This will be clearly evident from the derivation of (1) we will present now.

So for any weakly nonlinear dynamic system we have its enough good description in the form

$$x_o(t) = x_o^{(1)}(t) + x_o^{(2)}(t) + x_o^{(3)}(t) = \int_{-\infty}^{\infty} h^{(1)}(\tau) x_i(t-\tau) d\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(\tau_1,\tau_2) x_i(t-\tau_1) x_i(t-\tau_2) d\tau_1 d\tau_2 + \\ + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(3)}(\tau_1,\tau_2,\tau_3) x_i(t-\tau_1) x_i(t-\tau_2) x_i(t-\tau_3) d\tau_1 d\tau_2 d\tau_3 \tag{5}$$

obtained from (4). Note that to be consistent with the input-output notation assumed in (1), we have introduced the following: $x_o = y$ and $x_i = x$ in (5).

Equating to each other the expressions of the same order on both sides of (5) and taking afterwards the one-, two-, and three-dimensional Fourier transforms [5], respectively, on both sides of the resulting equations for the corresponding orders, we get

$$X_o^{(1)}(f) = H^{(1)}(f) X_i(f) \tag{6a}$$

$$X_o^{(2)}(f_1,f_2) = H^{(2)}(f_1,f_2) X_i(f_1) X_i(f_2) \tag{6b}$$

$$X_o^{(3)}(f_1,f_2,f_3) = H^{(3)}(f_1,f_2,f_3) X_i(f_1) X_i(f_2) X_i(f_3) \ . \tag{6c}$$

The sets of frequency variables: $\{f\}$, $\{f_1,f_2\}$, and $\{f_1,f_2,f_3\}$ occurring in (6a), (6b), and (6c), respectively, form the corresponding one-, two-, and three-dimensional frequency spaces. Obviously, they do not mean, in the above expressions, some concrete frequencies of harmonic signals (opposite to $f = \omega/(2\pi)$ in (1) being the frequency of a harmonic signal). Furthermore, $X_o^{(1)}(f)$, $X_o^{(2)}(f_1,f_2)$, and $X_o^{(3)}(f_1,f_2,f_3)$ in (6a), (6b), and (6c) are the one-, two-, and three-dimensional, respectively, Fourier transforms of the corresponding first-order $x_o^{(1)}(t)$, second-order $x_o^{(2)}(t)$, and third-order $x_o^{(3)}(t)$ partial responses determined in (5). Similarly,

$H^{(1)}(f)$, $H^{(2)}(f_1,f_2)$, and $H^{(3)}(f_1,f_2,f_3)$ are the corresponding Fourier transforms of $h^{(1)}(\tau)$, $h^{(2)}(\tau_1,\tau_2)$, and $h^{(3)}(\tau_1,\tau_2,\tau_3)$. They are also called the system nonlinear transfer functions of the first (linear one), of the second, and of the third order, respectively. Moreover, $X_1^{(1)}(f_x)$ with f_x meaning $f_1 = f$ or f_2 or f_3, denotes the ordinary (one-dimensional) Fourier transform of the input signal.

An useful version of the inverse n-dimensional (n=1,2,3, ..) Fourier transform is defined in [5]. It has the following form:

$$x_o^{(n)}(t) = \int_{-\infty}^{\infty} .. \int_{-\infty}^{\infty} X_o^{(n)}(f_1,..,f_n)\exp(j2\pi(f_1 t + .. + f_n t))df_1 \cdot\cdot df_n \tag{7}$$

Applying (7) in (6a), (6b), and (6c) gives

$$x_o^{(1)}(t) = \int_{-\infty}^{\infty} H^{(1)}(f)X_i(f)\exp(j2\pi ft)df \tag{8a}$$

$$x_o^{(2)}(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H^{(2)}(f_1,f_2)X_i(f_1)X_i(f_2)\exp(j2\pi(f_1 t + f_2 t))df_1 df_2 \tag{8b}$$

$$x_o^{(3)}(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H^{(3)}(f_1,f_2,f_3)X_i(f_1)X_i(f_2)X_i(f_3)\exp(j2\pi(f_1 t + f_2 t + f_3 t))df_1 df_2 df_3 \ . \tag{8c}$$

So the components $x_o^{(1)}(t)$, $x_o^{(2)}(t)$, and $x_o^{(3)}(t)$ occurring in (5) can be equivalently expressed through the system nonlinear transfer functions (of the corresponding orders) using (8a), (8b), and (8c).

The latter expressions can be further simplified (in sense of specializing for) for a particular harmonic input signal $x_i(t)$ of the form

$$x_i(t) = A_i \exp(j2\pi f_o t) \Leftrightarrow X_i(f) = A_i\delta(f - f_o) \tag{9}$$

where the relation with its ordinary (one-dimensional) Fourier transform $X_i(f)$ is shown, too. A_i in (9) is a real number and means the amplitude of this harmonic signal, but $f_o = \omega_o/(2\pi)$ means its frequency (and ω_o its angular frequency). Furthermore, δ means the Dirac impulse and f is the current frequency in Fourier transform (as in (6a)).

Introducing $X_i(f)$ given by the most right-hand side expression in (9) into (8a), (8b), and (8c), we obtain successively

$$x_o^{(1)}(t) = A_i \int_{-\infty}^{\infty} H^{(1)}(f)\delta(f - f_o)\exp(j2\pi ft)df \tag{10a}$$

$$x_o^{(2)}(t) = A_i^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H^{(2)}(f_1, f_2)\delta(f_1 - f_o)\delta(f_2 - f_o)\exp(j2\pi(f_1t + f_2t))df_1df_2 \tag{10b}$$

$$x_o^{(3)}(t) = A_i^3 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H^{(3)}(f_1, f_2, f_3)\delta(f_1 - f_o)\delta(f_2 - f_o)\delta(f_3 - f_o)\exp(j2\pi(f_1t + f_2t + f_3t))df_1df_2df_3 \quad . \tag{10c}$$

In (10b) and (10c), multiplications of the Dirac impulses occur under integral symbols. As shown in [4], [6], [7], and [8], they can be used in this form in the Volterra series. Therefore, using the sifting property of the Dirac impulse, not only in (10a), but also in (10b) and (10c), we get

$$x_o^{(1)}(t) = A_i H^{(1)}(f_o)\exp(j2\pi f_o t) = H^{(1)}(f_o)x_i(t) \tag{11a}$$

$$x_o^{(2)}(t) = A_i^2 H^{(2)}(f_o, f_0)\exp(j2\pi 2 f_o t) = H^{(2)}(f_o, f_0)\left(x_i(t)\right)^2 \tag{11b}$$

$$x_o^{(3)}(t) = A_i^3 H^{(3)}(f_o, f_o, f_o)\exp(j2\pi 3 f_o t) = H^{(3)}(f_o, f_o, f_o)\left(x_i(t)\right)^3 \tag{11c}$$

where $x_i(t)$ is given by the left-hand side expression in (9).

Note now that by introducing first $a_1(j\omega_o) = H^{(1)}(f_o)$, $a_2(j\omega_o) = H^{(2)}(f_o, f_0)$, and $a_3(j\omega_o) = H^{(3)}(f_o, f_o, f_o)$ with $\omega_o = 2\pi f_o$ in (11a), (11b), and (11c), respectively, taking next them into account in the most left-hand side equality of (5), and finally dropping for simplicity of notation the subscript small "o" by the angular frequency ω_o, we arrive exactly at the expression (1).

3 Nonlinear convolution and operation of taking real value do not commute

Consider first the linear (one-dimensional) convolution with the (linear) impulse response being a real function and the input signal being a complex function. Note that such the convolution and the operation of taking the real value of a complex number commute. That is the following

$$\int_{-\infty}^{\infty} h^{(1)}(\tau)\,\mathrm{Re}\{x_i(t-\tau)\}\,d\tau = \mathrm{Re}\left\{\int_{-\infty}^{\infty} h^{(1)}(\tau)x_i(t-\tau)d\tau\right\} \tag{12}$$

holds. In (12), $h^{(1)}(\tau)$ is assumed to be a real function of τ and $x_i(t-\tau)$ to be a complex function of $t-\tau$.

A customary usage of (12) in the theory of linear systems is a means of calculation of the response of a linear dynamic system to a real harmonic input signal like as $A_i\cos(\omega_o t)$ by applying the procedure sketched below

$$\int_{-\infty}^{\infty} h^{(1)}(\tau)A_i\cos\left(\omega_o(t-\tau)\right)d\tau = \int_{-\infty}^{\infty} h^{(1)}(\tau)\,\mathrm{Re}\left\{A_i\exp\left(j\omega_o(t-\tau)\right)\right\}d\tau = \mathrm{Re}\left\{\int_{-\infty}^{\infty} h^{(1)}(\tau)A_i\exp\left(j\omega_o(t-\tau)\right)d\tau\right\}. \tag{13}$$

That is by performing the operation of taking the real value of system response calculated for the corresponding complex signal of the form $A_i\exp(j\omega_o t)$.

Observe from (12) and (13) that the necessary and sufficient condition of correctness of the above way of calculation is that the operations: the linear system's one performed on the input signal (here, the linear convolution) and that of taking the real value of a complex number commute. This condition is not fulfilled in the case of nonlinear convolutions (that is of those associated with the nonlinear impulse responses $h^{(n)}(t_1,..,t_n)$ of orders $n \geq 2$).

For example, for the two-dimensional convolution, we have in general the following inequality

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(\tau_1,\tau_2)\,\mathrm{Re}\{x_i(t-\tau_1)\}\,\mathrm{Re}\{x_i(t-\tau_2)\}\,d\tau_1 d\tau_2 \neq \mathrm{Re}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(\tau_1,\tau_2)x_i(t-\tau_1)x_i(t-\tau_2)d\tau_1 d\tau_2\right\} \tag{14}$$

which, for the input signal $x_i(t) = A_i\exp(j\omega_o t)$, assumes the form

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(\tau_1,\tau_2)A_i^2\cos\left(\omega_o(t-\tau_1)\right)\cos\left(\omega_o(t-\tau_2)\right)d\tau_1 d\tau_2 \neq$$
$$\neq \mathrm{Re}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(\tau_1,\tau_2)A_i^2\exp\left[j\omega_o\left(\left(t-\tau_1\right)+\left(t-\tau_2\right)\right)\right]d\tau_1 d\tau_2\right\}=$$
$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(\tau_1,\tau_2)A_i^2\left[\cos\left(\omega_o(t-\tau_1)\right)\cos\left(\omega_o(t-\tau_2)\right)-\sin\left(\omega_o(t-\tau_1)\right)\sin\left(\omega_o(t-\tau_2)\right)\right]d\tau_1 d\tau_2 \ . \tag{15}$$

A consequence of lack of the commutation property of the operations mentioned above is the following: the response of a nonlinear system described by the Volterra series to the input signal in form of the real part of a complex signal can not be calculated as the real part of the response of this system to that complex signal. Another method must be applied and this is the subject of the next section.

4 Application of (8) to calculation of the response to input signal being a real function

The objective of this section is calculation of the response a system described by (5) driven by an input signal having the form $x_i\left(t\right)=A_i\cos\left(\omega_o t\right)$. To this end, note first that this input signal can be expressed in the following way

$$x_i\left(t\right)=\frac{A_i}{2}\left[\exp\left(j2\pi f_o t\right)+\exp\left(-j2\pi f_o t\right)\right]\Leftrightarrow X_i\left(f\right)=\frac{A_i}{2}\left[\delta\left(f-f_o\right)+\delta\left(f+f_o\right)\right] \tag{16}$$

where its Fourier transform is also given.

Furthermore, note that expressions (8) are general, not specialized for only some specific input signals (as (1)). So the input signal given by (16) can be applied in (8a), (8b) and (8c). More precisely, its Fourier transform can be introduced in these expressions.

First, after introduction in (8a), we get

$$x_o^{(1)}\left(t\right)=\frac{A_i}{2}\int_{-\infty}^{\infty}H^{(1)}\left(f\right)\left[\delta\left(f-f_o\right)+\delta\left(f+f_o\right)\right]\exp\left(j2\pi ft\right)df \ . \tag{17}$$

Exploiting then the sifting property of the Dirac impulse in (17) gives

$$x_o^{(1)}\left(t\right)=\frac{A_i}{2}\left[H^{(1)}\left(f_o\right)\exp\left(j2\pi f_o t\right)+H^{(1)}\left(-f_o\right)\exp\left(-j2\pi f_o t\right)\right]\cdot \tag{18}$$

And because the following

$$\left(H^{(1)}\left(f_o\right)\right)^* = \int_{-\infty}^{\infty} h^{(1)}\left(t\right)\left(\exp\left(-j2\pi f_o t\right)\right)^* dt = H^{(1)}\left(-f_o\right) \tag{19}$$

holds, (18) can be rewritten as

$$x_o^{(1)}\left(t\right) = \frac{A_i}{2}\left[H^{(1)}\left(f_o\right)\exp\left(j2\pi f_o t\right) + \left(H^{(1)}\left(f_o\right)\exp\left(j2\pi f_o t\right)\right)^*\right] = A_i \operatorname{Re}\left\{H^{(1)}\left(f_o\right)\exp\left(j2\pi f_o t\right)\right\} =$$
$$= \left|H^{(1)}\left(f_o\right)\right| A_i \cos\left(2\pi f_o t + \arg\left(H^{(1)}\left(f_o\right)\right)\right) . \tag{20}$$

The symbol $(\cdot)^*$ in (19) and (20) means the complex conjugate number (of a given complex number). Moreover, $\left|H^{(1)}\left(f_o\right)\right|$ and $\arg\left(H^{(1)}\left(f_o\right)\right)$ in (20) are the magnitude and phase, respectively, of the transfer function $H^{(1)}\left(f_o\right)$ at the frequency f_o.

Similarly, we get in the first step from (8b)

$$x_o^{(2)}\left(t\right) = \frac{A_i^2}{4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H^{(2)}\left(f_1, f_2\right)\left[\left(\delta\left(f_1 - f_o\right) + \delta\left(f_1 + f_o\right)\right)\left(\delta\left(f_2 - f_o\right) + \delta\left(f_2 + f_o\right)\right)\right]\exp(j2\pi(f_1 t + f_2 t))df_1 df_2 . \tag{21}$$

After performing multiplications of Dirac impulses occurring under the integral symbols in (21) and exploiting then the sifting property of these impulses, we arrive at

$$x_o^{(2)}\left(t\right) = \frac{A_i^2}{4}\left[H^{(2)}\left(f_o, f_o\right)\exp(j2\pi 2 f_o t) + 2H^{(2)}\left(f_o, -f_o\right)\exp(j2\pi\cdot 0\cdot t) + H^{(2)}\left(-f_o, -f_o\right)\exp\left(j2\pi\left(-2f_o\right)t\right)\right] . \tag{22}$$

In (22), the assumption that the nonlinear transfer function $H^{(2)}\left(f_1, f_2\right)$ is symmetric (for details on symmetry property of nonlinear transfer functions see, for example, [5]), that is in our case the following

$$H^{(2)}\left(f_o, -f_o\right) = H^{(2)}\left(-f_o, f_o\right) \tag{23}$$

holds, was used to shorten the notation. Note also that we can write

$$\left(H^{(2)}\left(f_o,f_o\right)\right)^* = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}\left(t_1,t_2\right)\left(\exp\left(-j2\pi\left(f_o t_1 + f_o t_2\right)\right)\right)^* dt_1 dt_2 = H^{(2)}\left(-f_o,-f_o\right) \tag{24}$$

similarly as (19). Using this result in (22) gives

$$x_o^{(2)}(t) = \frac{A_i^2}{2}H^{(2)}\left(f_o,-f_o\right) + \frac{A_i^2}{2}\left[H^{(2)}\left(f_o,f_o\right)\exp(j2\pi 2 f_o t) + \left(H^{(2)}\left(f_o,f_o\right)\exp(j2\pi 2 f_o t)\right)^*\right] = \frac{A_i^2}{2}H^{(2)}\left(f_o,-f_o\right) +$$
$$+\frac{A_i^2}{2}\operatorname{Re}\left\{H^{(2)}\left(f_o,f_o\right)\exp(j2\pi 2 f_o t)\right\} = \frac{A_i^2}{2}H^{(2)}\left(f_o,-f_o\right) + \frac{A_i^2}{2}\left|H^{(2)}\left(f_o,f_o\right)\right|\cos\left(2\cdot 2\pi f_o t + \arg\left(H^{(2)}\left(f_o,f_o\right)\right)\right) \ . \tag{25}$$

Finally, using (16) in (8c), we get

$$x_o^{(3)}(t) = \frac{A_i^3}{8}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H^{(3)}\left(f_1,f_2,f_3\right)\left[\left(\delta\left(f_1-f_o\right)+\delta\left(f_1+f_o\right)\right)\cdot\right.$$
$$\left.\cdot\left(\delta\left(f_2-f_o\right)+\delta\left(f_2+f_o\right)\right)\left(\delta\left(f_3-f_o\right)+\delta\left(f_3+f_o\right)\right)\right]\exp(j2\pi(f_1 t + f_2 t + f_3 t))df_1 df_2 df_3 \ . \tag{26}$$

And similarly as before, after performing multiplications of Dirac impulses occurring under the integral symbols in (26) and exploiting then the sifting property of these impulses, we arrive at

$$x_o^{(3)}(t) = \frac{A_i^3}{8}\left[H^{(3)}\left(f_o,f_o,f_o\right)\exp(j2\pi 3 f_o t) + 3H^{(3)}\left(f_o,f_o,-f_o\right)\exp(j2\pi f_o t) + 3H^{(3)}\left(-f_o,-f_o,f_o\right)\cdot\right.$$
$$\left.\cdot\exp(-j2\pi f_o t) + H^{(3)}\left(-f_o,-f_o,-f_o\right)\exp(-j2\pi 3 f_o t)\right] \ . \tag{27}$$

In (27), to shorten the notation, the symmetric nonlinear transfer functions were assumed. In particular, the following

$$H^{(3)}\left(f_o,f_o,-f_o\right) = H^{(3)}\left(f_o,-f_o,f_o\right) = H^{(3)}\left(-f_o,f_o,f_o\right) \tag{28a}$$

$$H^{(3)}\left(f_o,-f_o,-f_o\right) = H^{(3)}\left(-f_o,f_o,-f_o\right) = H^{(3)}\left(-f_o,-f_o,f_o\right) \tag{28b}$$

was applied.

Using the similar relations as (19) and (24) with regard to $H^{(3)}\left(-f_o,-f_o,f_o\right)$ and $H^{(3)}\left(-f_o,-f_o,-f_o\right)$ in (27), the latter can be rewritten as

$$x_o^{(3)}(t) = \frac{A_i^3}{8}\Big[H^{(3)}(f_o, f_o, f_o)\exp(j2\pi 3 f_o t) + 3H^{(3)}(f_o, f_o, -f_o)\exp(j2\pi f_o t) + $$
$$+3\left(H^{(3)}(f_o, f_o, -f_o)\exp(j2\pi f_o t)\right)^* + \left(H^{(3)}(f_o, f_o, f_o)\exp(j2\pi 3 f_o t)\right)^* \Big] \tag{29a}$$

or alternatively as

$$x_o^{(3)}(t) = \frac{3}{4}A_i^3 \operatorname{Re}\left\{H^{(3)}(f_o, f_o, -f_o)\exp(j2\pi f_o t)\right\} + \frac{A_i^3}{4}\operatorname{Re}\left\{H^{(3)}(f_o, f_o, f_o)\exp(j2\pi 3 f_o t)\right\} =$$
$$= \frac{3}{4}A_i^3 \left|H^{(3)}(f_o, f_o, -f_o)\right| \cos\left(2\pi f_o t + \arg\left(H^{(3)}(f_o, f_o, -f_o)\right)\right) +$$
$$+ \frac{A_i^3}{4}\left|H^{(3)}(f_o, f_o, f_o)\right| \cos\left(3\cdot 2\pi f_o t + \arg\left(H^{(3)}(f_o, f_o, f_o)\right)\right) \quad . \tag{29b}$$

Finally, note that the sum of the expressions staying after the last equality symbol: in (20), in (25) (after neglecting the dc component), and in (29b) (after neglecting the compression component at frequency f_o) constitutes the leading formula in [1] (eq. 10 there) and [2] (eq. 6 there).

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POPRAWNOŚĆ WZORÓW PODSTAWOWYCH UŻYTYCH W PEWNEJ UPROSZCZONEJ METODZIE OBLICZEŃ ZNIEKSZTAŁCEŃ HARMONICZNYCH SCALONYCH UKŁADÓW ANALOGOWYCH

Streszczenie – W pracy pokazano, że sposób obliczeń zniekształceń harmonicznych opracowany przez Palumbo i Pennisiego jest niczym innym tylko uproszczoną analizą z wykorzystaniem szeregu Volterry. Ponadto wykazano, że użyty przez nich opis układu za pomocą pewnego szeregu potęgowego nie ma walorów ogólności (zakres jego stosowalności jest ograniczony tylko do pewnej postaci sygnałów wejściowych). Przedstawiono również właściwe interpretacje założeń i poprawne wyprowadzenia podstawowych zależności użytych przez Palumbo i Pennisiego.