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MATHEMATICAL MODELING OF AEROBIC WASTEWATER TREATMENT IN POROUS MEDIUM

Summary – A mathematical model of aerobic wastewater treatment in a porous medium, which taking into account the interaction of bacteria, organic and biological inoxidizable substances was built. An algorithm of the model of nonlinear singularly perturbed tasks of the type "filtration-convection-diffusion-mass-transfer" was proposed. A computer experiment based on it was done

Keywords: filtration, model of aerobic wastewater treatment, asymptotics, nonlinear tasks

1 Introduction

The methods of biological wastewater treatment have several significant advantages than other methods due to excellent ability of microorganisms to adapt in very unfavorable situations (high concentrations and toxicity, the complex mixture of pollutants) [1-11].

Searching cost-effective and environmentally acceptable methods of treatment of industrial and domestic wastewater still remain highly relevant. Constantly increasing demands for the purity of the environment, restoring water resources requirement and its rational use cause continuous search for improved, reliable, inexpensive and effective technical solutions for wastewater treatment.

For wastewater treatment of low and medium productivity in the technological schemes of wastewater treatment biofilters became widely employed [10]. Simple in constructive terms, as fastidious during the exploitation they satisfy the requirements of various users in the purification of the varied of property and regime inflow of wastewater [11].

For description of their work was proposed a numeric mathematical models which focused on the description of the processes which occurring in biofilm which is formed on the surface of the biofilter filler and its thickness. There were was established working conditions of the filter in the initial period and in period of long exploitation [9]. However, the description of complex interactions in the treatment of many factors that vary over time scientists was paid a small attention [1, 3, 8]. Many mathematical models which describe the process of sewage treatment in the biofilters show a growing interest of researchers to this problem and the imperfection of their proposed models.

The aim of this work is to develop the mathematical model which allows to calculate the concentrations of biofilm, organic and biologically inoxidizable substances in porous medium.

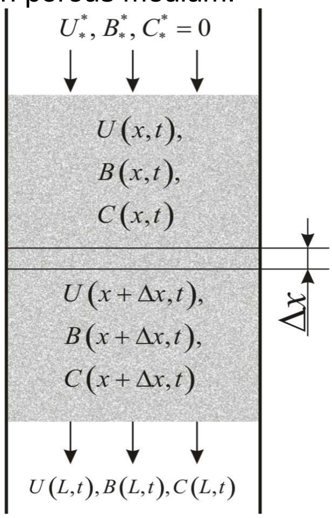

Fig. 1. Schematic representation of the process of aerobic treatment

2 Setting a task

The processes which occurring during the purification of fluid from the organic pollutions are considered. Based on existing literature [1-6] and experimental data were considered the following processes - a decomposition of organic substances, growth and death of biofilm, dynamics of change in the concentration of biofilm processes including sorption and desorption and diffusion (Fig.1).

For describing the dynamic of microbial populations we using the classical Monod equation which take into account dying cells of microorganisms [1, 4, 6]:

$$\frac{\partial B}{\partial t} = \frac{\mu_{\max} U}{U + K_S} B - \lambda B \,, \tag{1}$$

where B - the concentration of biomass; $\mu_{\max}$ - the maximum rate of biofilm growth; U - concentration of pollution in the fluid; K_S - a constant of affinity substrate for microorganism; λ - the rate of mortality of the biofilm.

Taking into account that the front of concentration of biofilm moves together with the contaminated substance in a porous medium, we got the following equation for the growth, mortality and transfer of bacterial biofilm:

$$\frac{\partial B}{\partial t}=\frac{\mu_{\max}U}{U+K_S}B-\lambda B-\frac{v}{\sigma_e}\frac{\partial B}{\partial x} \tag{2}$$

where v - rate of substance; σ_e - effective porosity ($\sigma_e=k_a\rho+\sigma$, k_a - coefficient of bacteria adsorption; ρ - medium density; σ - porosity).

Decreasing the amount of contamination of fluid is due to decomposition by bacteria and filtration of biological inoxidizable substances. Biological decomposition can be described by Monod equation and washing out – by equation of convective transfer, their combination describes the change in concentration of pollution with time:

$$\frac{\partial U}{\partial t}=-\frac{1}{q}\frac{\mu_m U}{U+K_S}B-v\frac{\partial U}{\partial x}-\frac{v\sigma}{\rho}\frac{\partial C}{\partial x}+D_U\frac{\partial^2 U}{\partial x^2} \tag{3}$$

where q - proportionality coefficient that links the amount of cells which formed during absorbing by substrate; C - concentration of biological inoxidizable substance; D_U - diffusion coefficient, $D_U=b_U\varepsilon$, $0<b_U\le 1$.

In most cases we can assume that the bacteria produce surfactants simultaneously with the decomposition of hydrocarbons, namely, some amount of hydrocarbons use for creating of the surfactants.

Similarly to the previous, change of the concentration of biological inoxidizable substance takes place according to Monod equation :

$$\sigma\frac{\partial C}{\partial t}=\frac{1}{\rho q_S}\frac{\mu_m U}{K_S+U}B-v\frac{\partial C}{\partial x}-\beta C+D_C\frac{\partial^2 C}{\partial x^2} \tag{4}$$

where q_S - proportionality coefficient that links the amount of maked organically inoxidizable substance with absorbed substrate; β - coefficient which characterizes the amount of trapped particles of biological inoxidizable substances by filter; D_C - diffusion coefficient, $D_C=b_C\varepsilon$, $0<b_C\le 1$, ε - small parameter.

The equation (2), (3) and (4) form a system which describe the change in concentration of bacteria, organic substance and biological inoxidizable substance in the porous medium. Based on the above we consider the following model problem rewritten as follows:

$$\begin{cases} \dfrac{\partial B}{\partial t} = \Phi(U)B - \lambda B - \dfrac{v}{\sigma_e}\dfrac{\partial B}{\partial x}, \\ \dfrac{\partial U}{\partial t} = -u\Phi(U)B - v\dfrac{\partial U}{\partial x} - \dfrac{v\sigma}{\rho}\dfrac{\partial C}{\partial x} + D_U\dfrac{\partial^2 U}{\partial x^2}, \\ \sigma\dfrac{\partial C}{\partial t} = c\Phi(U)B - v\dfrac{\partial C}{\partial x} - \beta C + D_C\dfrac{\partial^2 C}{\partial x^2}, \end{cases} \tag{5}$$

$$\begin{gathered} B\big|_{x=0} = B_*^*(t),\ U\big|_{x=0} = U_*^*(t),\ C\big|_{x=0} = 0, \\ \left.\frac{\partial U}{\partial x}\right|_{x=L} = 0, \left.\frac{\partial C}{\partial x}\right|_{x=L} = 0, \\ B_{t=0} = 0,\ C\big|_{t=0} = 0, U\big|_{t=0} = 0, \end{gathered} \tag{6}$$

where $\Phi(U) = \dfrac{\mu_{\max}U}{U + K_S}$, L - the filter length, $u = q_*^{-1}\varepsilon$, $c = (\rho q_{S*})^{-1}\varepsilon$ ($q_* = q\varepsilon,\ q_{S*} = q_S\varepsilon$).

3 The algorithm of solution

The obtained solution the problem (1) - (2) with precision $O\left(\varepsilon^{n+1}\right)$ is looking as asymptotic rows in degree of small parameter ε [7, 8]

$$B(x,t) = B_0(x,t) + \sum_{i=1}^{n}\varepsilon^i B_i(x,t) + R_B(x,t,\varepsilon),$$

$$U(x,t) = U_0(x,t) + \sum_{i=1}^{n}\varepsilon^i U_i(x,t) + \sum_{i=0}^{n}\varepsilon^i \tilde{U}_i\left(\tilde{\xi},t\right) + R_U(x,t,\varepsilon), \tag{7}$$

$$C(x,t) = C_0(x,t) + \sum_{i=1}^{n}\varepsilon^i C_i(x,t) + \sum_{i=0}^{n}\varepsilon^i \tilde{C}_i(\tilde{\mu},t) + R_C(x,t,\varepsilon),$$

where R_B, R_U, R_C – the remaining members, $B_i(x,t)$, $U_i(x,t)$, $C_i(x,t)$ ($i = \overline{0,n}$) – the regular members of asymptote, $\tilde{U}_i\left(\tilde{\xi},t\right), \tilde{C}_i(\tilde{\mu},t)$ ($i = \overline{0,n}$) – function of cross-border layer type (corrections on the output filtration substances), $\tilde{\xi} = (L-x)\cdot\varepsilon^{-1}$, $\tilde{\mu} = (L-x)\cdot\varepsilon^{-1}$ – relevant regulation transformation.

As a result of substitution (7) in system (1) - (2) and using the standard "procedure of equalization" [7, 8] we obtain the such tasks to determine the functions, $B_i(x,t)$, $U_i(x,t)$, $C_i(x,t)$ ($i = \overline{0,n}$):

$$\begin{cases} \dfrac{\partial B_0}{\partial t} = \left(\Phi\left(U_0\right) - \lambda\right) B_0 - \dfrac{v}{\sigma_e} \dfrac{\partial B_0}{\partial x}, \\ \dfrac{\partial U_0}{\partial t} + v \dfrac{\partial U_0}{\partial x} = -\dfrac{v\sigma}{\rho} \dfrac{\partial C_0}{\partial x}, \\ \sigma \dfrac{\partial C_0}{\partial t} + v \dfrac{\partial C_0}{\partial x} + \beta C_0 = 0, \\ B\big|_{x=0} = B_*^*(t),\ U\big|_{x=0} = U_*^*(t),\ C\big|_{x=0} = 0,\ B\big|_{t=0} = 0, C\big|_{t=0} = 0, U\big|_{t=0} = 0, \end{cases} \tag{8}$$

$$\begin{cases} \dfrac{\partial B_i}{\partial t} = \Phi\left(\sum_{i=1}^{n} U_i\right) B_i - \lambda B_i - \dfrac{v}{\sigma_e} \dfrac{\partial B_i}{\partial x}, \\ \dfrac{\partial U_i}{\partial t} + v \dfrac{\partial U_i}{\partial x} = -\dfrac{v\sigma}{\rho} \dfrac{\partial C_i}{\partial x} - \dfrac{1}{q_*} \Phi\left(\sum_{i=1}^{n} U_{i-1}\right) B_{i-1} + b_U \dfrac{\partial^2 U_{i-1}}{\partial x^2}, \\ \sigma \dfrac{\partial C_i}{\partial t} + v \dfrac{\partial C_i}{\partial x} + \beta C_i = \dfrac{1}{\rho q_{S*}} \Phi\left(\sum_{i=1}^{n} U_{i-1}\right) B_{i-1} + b_C \dfrac{\partial^2 C_{i-1}}{\partial x^2}, \\ B\big|_{x=0} = 0, U\big|_{x=0} = 0,\ C\big|_{x=0} = 0,\ B\big|_{t=0} = 0, U\big|_{t=0} = 0,\ C\big|_{t=0} = 0. \end{cases} \tag{9}$$

As a result of their decision we got:

$C_0\left(x,t\right) = 0$

$$C_0\left(x,t\right) = 0\ ,\ U_0\left(x,t\right) = \begin{cases} \int_0^t g_0\left(x - v\left(t - \tilde{t}\right), \tilde{t}\right) d\tilde{t},\ t \le \dfrac{x}{v}, \\ \dfrac{1}{v} \int_0^x g_0\left(\tilde{x}, \dfrac{1}{v}\left(\tilde{x} - x + vt\right)\right) d\tilde{x} + U_*^*\left(t - \dfrac{x}{v}\right),\ t > \dfrac{x}{v}, \end{cases} \tag{10}$$

$$B_0\left(x,t\right) = \begin{cases} B_*^*\left(t - \dfrac{\sigma_e x}{v}\right) \cdot e^{\frac{\left(\lambda - \Phi\left(U_*^*\right)\right)\sigma_e x}{v}},\ t > \dfrac{\sigma_e x}{v}, \\ 0,\ t \le \dfrac{\sigma_e x}{v}, \end{cases} \tag{11}$$

$$C_i\left(x,t\right) = \begin{cases} \dfrac{\sigma}{v} e^{\frac{\beta\sigma x}{v}} \cdot \int_0^x e^{-\frac{\beta\sigma\tilde{x}}{v}} W_i\left(\tilde{x}, t + \dfrac{\sigma}{v}\left(\tilde{x} - x\right)\right) d\tilde{x},\ t > \dfrac{\sigma x}{v}, \\ \dfrac{e^{\beta t}}{\sigma} \cdot \int_0^t e^{-\beta\tilde{t}} W_i\left(\dfrac{v}{\sigma}\left(\tilde{t} - t\right) + x, \tilde{t}\right), t \le \dfrac{\sigma x}{v}, \end{cases} \tag{12}$$

$$U_i(x,t) = \begin{cases} \int_0^t g_i\left(x - v(t-\tilde{t}), \tilde{t}\right) d\tilde{t}, \; t \le \frac{x}{v}, \\ \frac{1}{v}\int_0^x g_i\left(\tilde{x}, \frac{1}{v}(\tilde{x} - x + vt)\right) d\tilde{x}, \; t > \frac{x}{v}, \end{cases} \qquad B_i(x,t) = 0 \qquad (13)$$

where

$g_0(x,t) = \frac{v}{\rho}\frac{\partial C_0}{\partial x}$, $g_i = -\frac{v\sigma}{\rho}\frac{\partial C_i}{\partial x} - \frac{1}{q_*}\Phi\left(\sum_{i=1}^{n} U_{i-1}\right) b_{i-1} + b_U \frac{\partial^2 U_{i-1}}{\partial x^2}$,

$W_i = \frac{1}{\rho q_{S*}}\Phi\left(\sum_{i=1}^{n} U_{i-1}\right) B_{i-1} + b_C \frac{\partial^2 C_{i-1}}{\partial x^2}$.

The functions $\tilde{U} = \sum_{i=0}^{n} \tilde{U}_i \varepsilon^i$, $\tilde{C} = \sum_{i=0}^{m} \tilde{C}_i \varepsilon^i$ which is assigned for the removal of inconsistencies, which were brought by the built regular parts, $U(x,t) = \sum_{i=0}^{n} U_i \varepsilon^i$, $C(x,t) = \sum_{i=0}^{n} C_i \varepsilon^i$ at around point $x = L$ (output of filtration flow), that is provide implementation of condition: $\frac{\partial}{\partial x}\left(U + \tilde{U}\right) = O\left(\varepsilon^{n+1}\right)$, $\frac{\partial}{\partial x}\left(C + \tilde{C}\right) = O\left(\varepsilon^{n+1}\right)$. We are have proper task analogical to [7, 8] for the searching this functions and estimation of remaining members.

4 The results of numerical calculations

There are present results of calculations by formulas (7) when $B_*^*(t) = 10$ *kl/ml*, $U_*^*(t) = 8$ *mg/l*, $v = 5\ m/hour$, $\beta = 36^{-1}$ *m²/s*, $\lambda = 0.06\ \partial o \delta^{-1}$, $\sigma_e = 5$, $\sigma = 0.37$, $\mu_m = 2.5\ \mathrm{day}^{-1}$, $K_S = 0.1$ *mg/l*, $b_U = 1.25 \cdot 10^{-4}$ *m²/hour*, $b_C = 2 \cdot 10^{-4}$ *m²/hour*, $\rho = 1.5\ g/sm^3$, $q = 2 \cdot 10^{-9}$ *kl/g*, $q_S = 4 \cdot 10^{11}$ *kl/g*.

In Figures 2-3 presents the results of modeling that describe the process of biological treatment in the filter. In Figure 2 is shown the distribution of pollution concentration along the filter at different times, in Figure 3 - distribution of pollution concentration at the filter exit, in Figure 4 - distribution of concentration of bacteria along the filter.

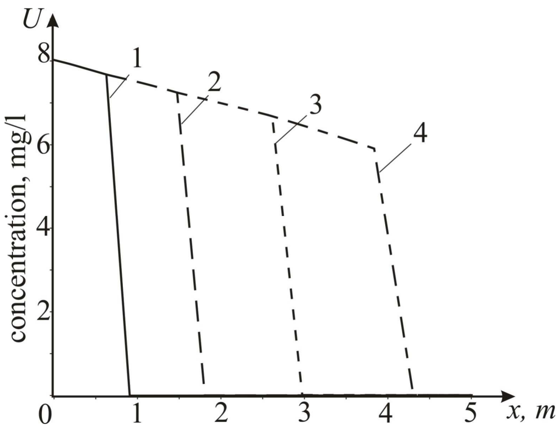

Fig. 2. Distribution of concentration of pollution concentration along the filter at the moment time $t_1 = 10\ hour$, $t_2 = 20\ hour$, $t_3 = 30\ hour$, $t_4 = 40\ hour$.

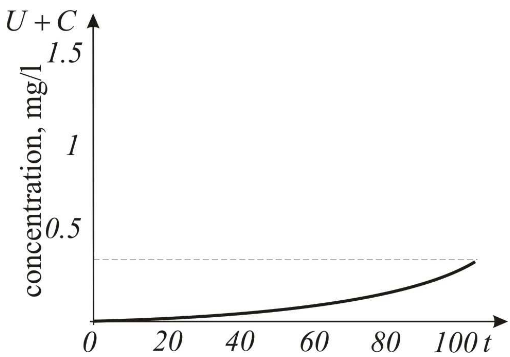

Fig. 3. Distribution of the pollution concentration at the outlet of the filter

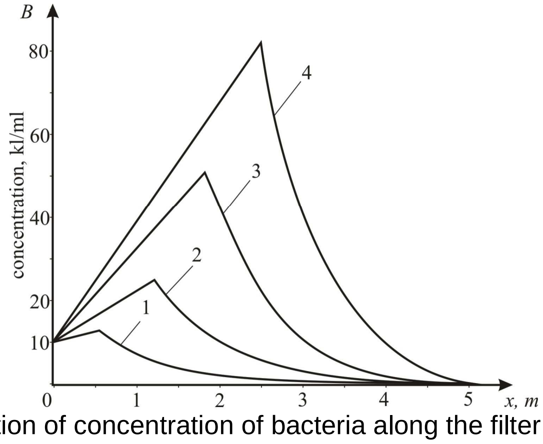

Fig. 4. Distribution of concentration of bacteria along the filter at the moment time $t_1 = 10\ hour$, $t_2 = 20\ hour$, $t_3 = 30\ hour$, $t_4 = 40\ hour$

5 Conclusion

Application of mathematical modeling in this case allow to estimate the effective time of the biofilter. The obtained formulas and graphical dependencies between values are effective for theoretical studies which aimed at optimizing the parameters of filtering process (time of protective action download, filter sizes, etc.).

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